JOURNAL OF PROPULSION AND POWER
Vol. 13, No. 5, September–October 1997

Theoretical Formulation for Sauter Mean Diameter of Pressure-Swirl Atomizers

H. S. Couto,* J. A. Carvalho Jr.,* and D. Bastos-Netto†
National Space Research Institute, 12630-000, Cachoeira Paulista, SP, Brazil

This work derives a general formula for determining the Sauter mean diameter of sprays generated by pressure-swirl atomizers. This is done by extending the theory of the aerodynamic instability and disintegration of viscous liquid flat sheets to describe the dynamic behavior of the hollow conical sheet generated by these atomizers. The derived theoretical equation for the conical spray droplets mean diameter includes atomizer geometrical parameters as well as liquid fuel and atomizing gas flow parameters. The results compare well with several existing empirical and semiempirical correlation formulas and available experimental data.

Introduction

PRESSURE-SWIRL atomizers have widespread use among systems requiring a quick dispersion of liquids in a gaseous environment, because they are simple and reliable, present good atomization characteristics, and have low pumping power requirements.[1] Although being the target of several investigations,[1–5] these atomizers still lack a reliable expression for the Sauter mean diameter (SMD) of the droplets in the spray they generate, and even the existing empirical correlations look far from presenting the reasonable uniformity expected from a universal expression.[1]

Although the liquid flat sheet formation and breakup mechanisms have been understood for quite a while,[6–8], this is not so for the conical shaped sheet generated by pressure-swirl atomizers, which needs a theory to properly describe its breakup and droplet formation dynamics.[1,9]

According to Lefebvre[9] and Eisenklam,[10] the theory of attenuating conical sheets in hollow cone pressure-swirl atomizers had yet to be developed. York et al.[11] used an infinite flat sheet model in their theoretical analysis, and were able to make only a rough estimate of the size of the drops produced by a pressure-swirl nozzle. Also, Wang and Lefebvre[1] stated that because of the highly complex nature of the atomization process in pressure-swirl atomizers, most workers have resorted to various empirical correlation formulas, which displayed sufficient disparity to preclude any possibility of a universal correlation, probably because of their use of widely different fluids and operating conditions in their experiments. The main contribution of this paper is, under the assumption of a hollow cone liquid sheet, the development of a theoretical general formula to describe pressure-swirl atomizers, which holds for fluids within wide ranges of viscosity and surface tension.

The work of Dombrowski and Johns[6] on fan spray flat liquid sheets is reviewed, showing the presence of the Ohnesorge number in the calculation of the ligament diameter and recovering Rayleigh's results on the size of a droplet formed from a ligament collapse. This paper then extends those results to the hollow conical sheet produced by a pressure-swirl nozzle by recalling that the wavelength of any perturbation (i.e., any ripple) of the conical sheet should be much smaller than its characteristic length. This leads to an expression for droplets diameter in the hollow conical spray that compares well with several existing empirical and semiempirical correlation formulas and with available experimental data.

Theoretical Background

The way a thinning plane viscous liquid sheet, formed by a fan-spray atomizer, disintegrates into fragments that then contract by surface tension, thus forming unstable ligaments and breaking into droplets, has been thoroughly discussed by Dombrowski and Johns,[6] who found the following expression for the diameter of those ligaments d_L (cm), at the sheet breakdown mode:

$$d_L = \left(\frac{8}{9}\right)^{1/3}\left(\frac{k^2\sigma^2}{\rho_a\rho_L U^2}\right)^{1/6}\left[1 + 2.6\mu_L\left(\frac{k\rho_a^4U^8}{72\rho_L^2\sigma^5}\right)^{1/3}\right]^{1/5} \tag{1}$$

where σ (dyne/cm) is the liquid surface tension, μ_L (cp) is the liquid dynamic viscosity, ρ_a (g/cm^3) is the density of the surrounding medium (assumed gaseous and quiescent) at pressure p_a (MPa), ρ_L (g/cm^3) is the density of the liquid, U (cm/s) is the velocity of the radiating sheet, and k is a constant obtained by choosing a hyperbolic relationship between the sheet thickness h and the time t; i.e., $ht = k$. The following expression was obtained for the droplet diameter d_d (cm):

$$d_d = \left(\frac{3\pi}{\sqrt{2}}\right)^{1/3} d_L\left[1 + \frac{3\mu_L}{(\rho_L\sigma d_L)^{1/2}}\right]^{1/6} \tag{2}$$

Additionally, Eq. (2) can be written as

$$d_d = 1.88 d_L(1 + 3Oh)^{1/6} \tag{3}$$

where $1.88d_L$ is very close to the expression found by Rayleigh[9] for fan sheets ($1.89d_L$). The term $(1 + 3Oh)^{1/6}$ is a correction to Rayleigh's theory, where

$$Oh = \frac{\mu_L}{(\sigma\rho_L d_L)^{1/2}} \tag{4}$$

is the Ohnesorge number.[9]

In the present case of a pressure-swirl atomizer, the liquid is forced through a swirler prior to leaving the nozzle, thus acquiring enough angular momentum so that an air-cored vortex is established and, upon emerging from the nozzle with both nonzero tangential and axial velocity components, pro-

Received Aug. 27, 1996; revision received April 30, 1997; accepted for publication April 30, 1997.

*Senior Researcher, Combustion and Propulsion Associate Laboratory.

†Senior Researcher, Combustion and Propulsion Associate Laboratory. Associate Fellow AIAA.

duces a hollow conical thin sheet that rapidly disintegrates into fragments and then into droplets.[1]

Basic Assumptions

The theory developed for thin plane liquid sheets can be used in the problem of thin conical liquid sheets if the following is assumed.

1) This conical sheet possesses a rupture radius much larger than its thickness. Experimental observation shows that the cone starts to breakup into ligaments at a few millimeters from the injection point, which is a distance at least one order of magnitude larger than the film thickness.

2) Once the conical sheet is established, the amplitude of any disturbance (ripple) away from the injector tip is much smaller than the cone diameter, so that the ripple sees the conical sheet as a plane sheet. This amplitude has a magnitude on the order of half of the film thickness.

3) The wavelength of any ripple formed in the liquid film grows until it has an amplitude equal to the radius of the ligament, so that one droplet will be produced per wavelength.[6]

Problem Description and Solution

The liquid is initially agglutinated and irradiates as a conical sheet from the nozzle tip (Fig. 1). For a plane sheet, the thickness at any section y from the injection point was presented by Dombrowski and Johns[6] as being $h = K_1/y$, where K_1 is a constant.

For a radiating conical sheet, the thickness at any section is considered to be given by $h = K_1/X$, where X is the coordinate

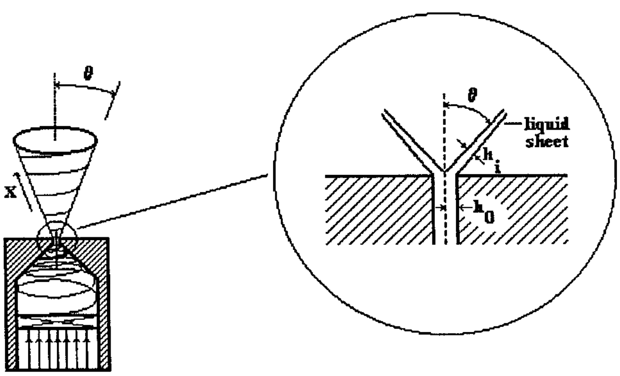

Fig. 1 Schematics of a pressure swirl nozzle.

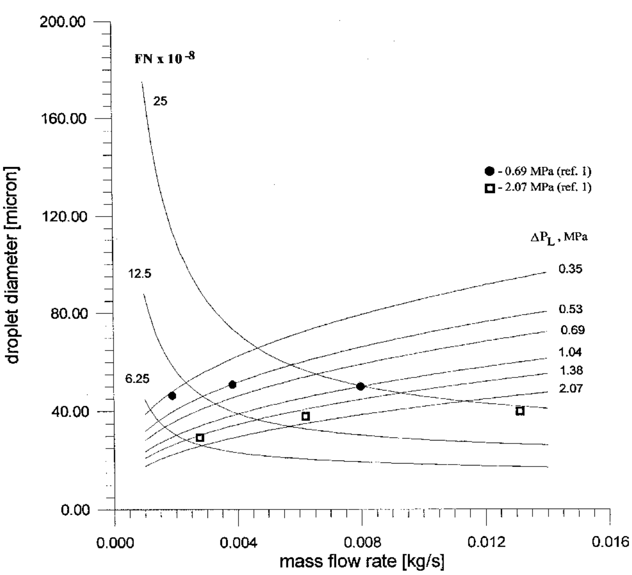

Fig. 2 Spray mean diameter vs liquid mass flow rate, for different injection pressures and flow numbers, P_A = 0.1 MPa (water; u = 60 deg).

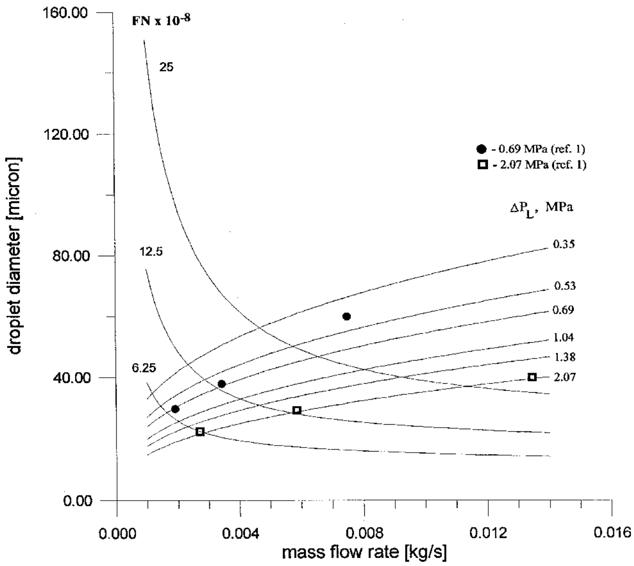

Fig. 3 Spray mean diameter vs liquid mass flow rate, for different injection pressures and flow numbers, P_A = 0.1 MPa (water; u = 90 deg).

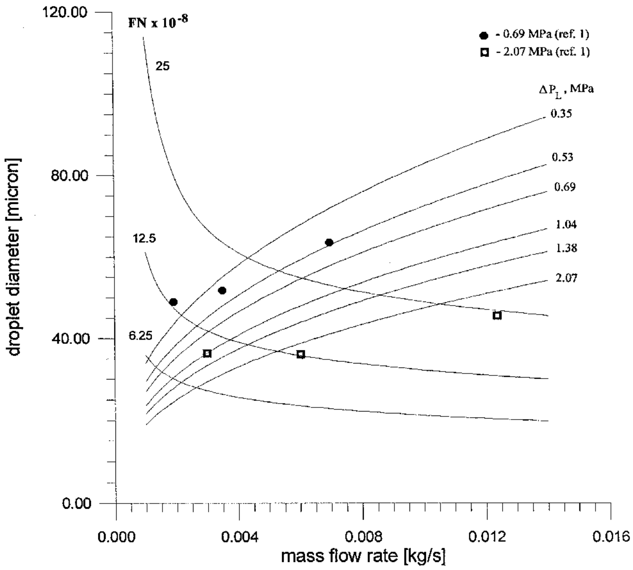

Fig. 4 Spray mean diameter vs liquid mass flow rate, for different injection pressures and flow numbers, P_A = 0.1 MPa (diesel, DF-2; u = 60 deg).

along the sheet. Dombrowski and Johns[6] used $K_2 = ht$, where K_2 is a constant and t is time. Therefore,

$$K_2 = K_1 t/X = K_1/U \tag{5}$$

The sheet develops in such a way that $hX = K_1$ = const. The only way for this to be achieved is that, at any coordinate $X = nh_i$ (where n is any positive number and h_i is the initial film thickness of the liquid sheet where it turns around to form the cone), the sheet thickness should be equal to h_i/n. Then

$$K_1 = (h_i/n)nh_i = h_i^2 \tag{6}$$

$$K_2 = h_i^2/U \tag{7}$$

This initial thickness h_i can be related to the film thickness within the final orifice h_0, by the expression $h_i = h_0 \cos\theta$, where θ is the cone semiangle.[1] The velocity U is given, for

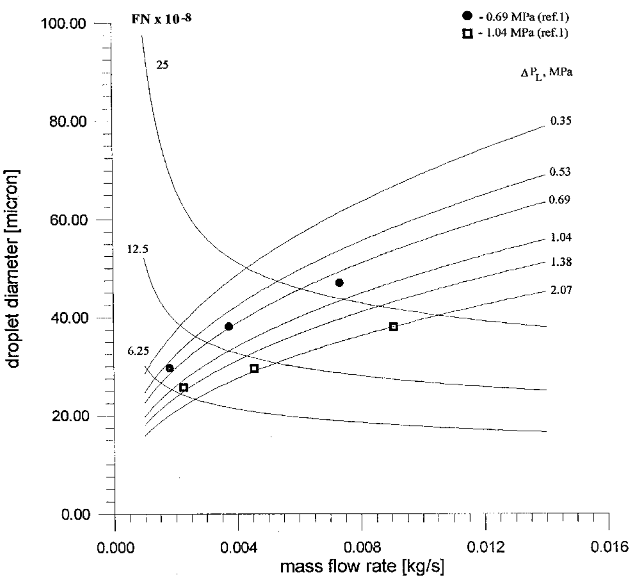

Fig. 5 Spray mean diameter vs liquid mass flow rate, for different injection pressures and flow numbers, P_A = 0.1 MPa (diesel, DF-2; θ = 90 deg).

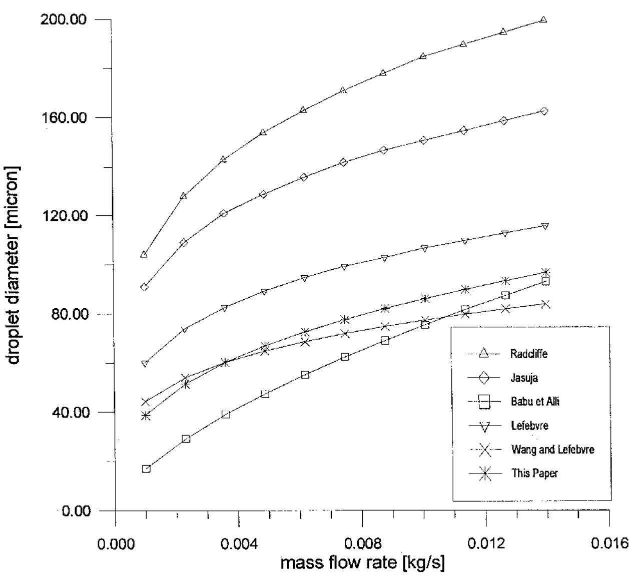

Fig. 6 Spray mean diameter vs liquid mass flow rate, for existing correlations (water; θ = 60 deg; Δp_L = 0.35 MPa).

the incompressible flow case, as $U = U_0/\cos\theta$, where U_0 is the velocity of the liquid at the atomizer tip.[9] Consequently,

$$K_2 = h_0^2 \cos^3\theta / U_0 \tag{8}$$

Inserting Eq. (8) into the equation derived by Dombrowski and Johns[6] for the ligament diameter in a plane sheet [Eq. (1)], the following equation is obtained for the rotating conical sheet:

$$d_L = 0.9615 \cos\theta \left(\frac{h_0^4 \sigma^2 \cos^2\theta}{U_0^4 \rho_a \rho_L}\right)^{1/6} \times \left[1 + 2.6\mu_L \cos\theta \left(\frac{h_0^2 \rho_a^4 U_0^7}{72\rho_L^2 \sigma^5 \cos^8\theta}\right)^{1/3}\right]^{0.2} \tag{9}$$

Instead of using the result of Dombrowski and Johns[6] for estimating the droplet diameter, one should recall that, in obtaining Eq. (2) those authors used Weber's results,[12] which were developed for the breaking of a liquid column into several droplets; viscosity played an important role in that process. However, this does not seem to be the case when the ligament collapses into a single spherical droplet as a result of the surface tension action. Therefore, assuming that the collapse of a ligament produces a droplet of diameter d_d, according to Rayleigh's mechanism,[9] one may write

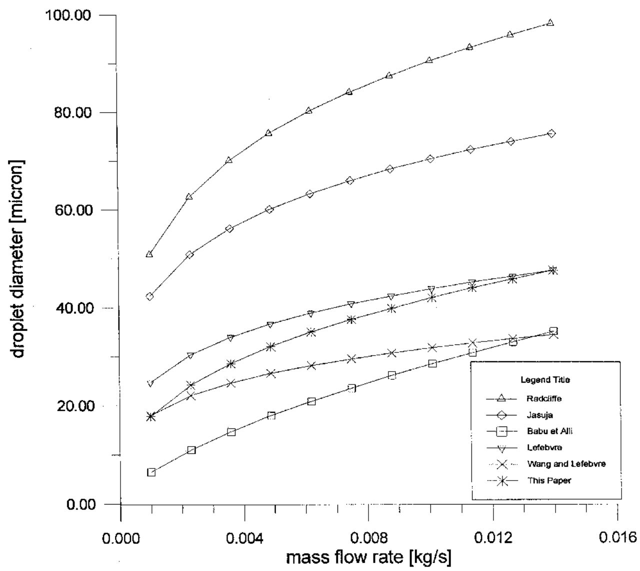

Fig. 7 Spray mean diameter vs liquid mass flow rate, for existing correlations (water; θ = 60 deg; Δp_L = 2.07 MPa).

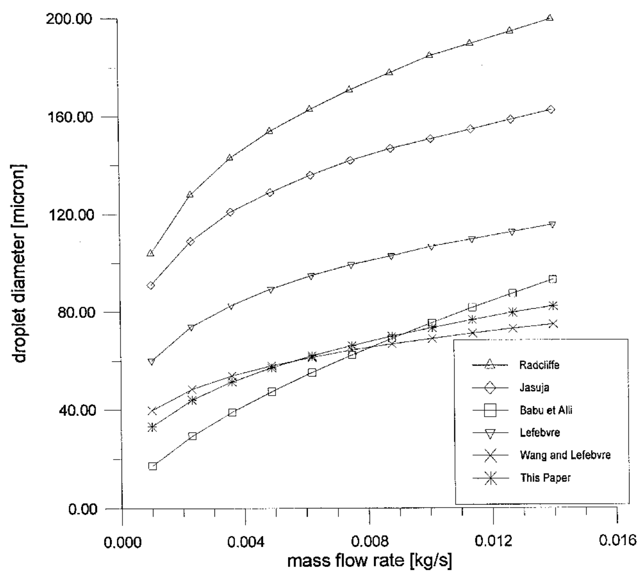

Fig. 8 Spray mean diameter vs liquid mass flow rate, for existing correlations (water; θ = 90 deg; Δp_L = 0.35 MPa).

$$d_d = 1.89 d_L \tag{10}$$

The flow number (FN) can be expressed by

$$\mathrm{FN} = \dot{m} / \sqrt{\rho_L \Delta p_L} \tag{11}$$

where $\dot{m}$ is the liquid mass flow rate, and Δp_L is the atomizer pressure differential. The spray cone half-angle θ has been shown by Rizk and Lefebvre[13] to depend on the air core size only, i.e.,

$$x = \frac{\sin^2\theta}{1 + \cos^2\theta} \tag{12}$$

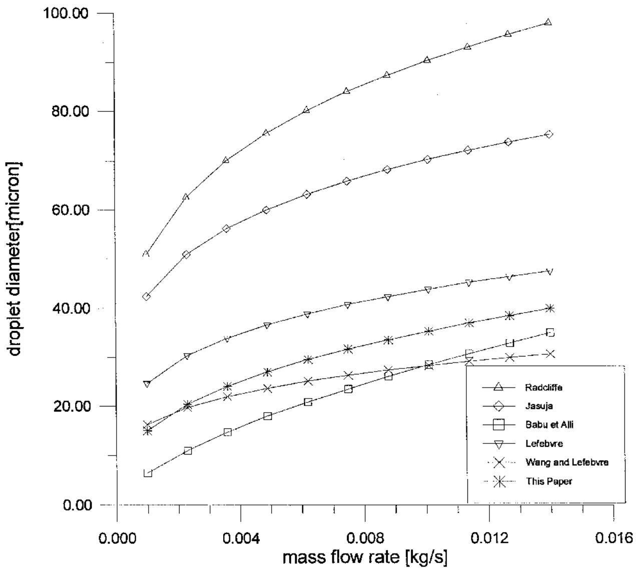

Fig. 9 Spray mean diameter vs liquid mass flow rate, for existing correlations (water; u = 90 deg; Dp_L = 2.07 MPa).

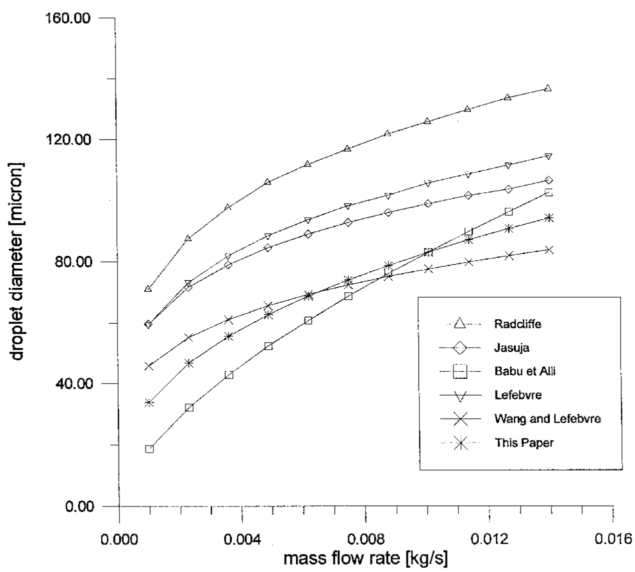

Fig. 10 Spray mean diameter vs liquid mass flow rate, for existing correlations (diesel; u = 60 deg; Dp_L = 0.35 MPa).

where x is the ratio between the air core area A_a and the discharge orifice area A_0. If not given in advance, the final discharge orifice diameter d_0 can be estimated by the expression

$$d_0 = 2\sqrt{\frac{\mathrm{FN}}{\sqrt{2}\pi(1-x)}} \tag{13}$$

The liquid velocity at the atomizer tip U_0 is given by

$$U_0 = \sqrt{2\Delta p_L/\rho_L} \tag{14}$$

and h_0, the film thickness at the tip can be estimated as[14]

$$h_0 = \frac{0.00805\sqrt{\rho_L}\mathrm{FN}}{d_0\cos\theta}\ \text{(MKS units)} \tag{15}$$

Results and Comparison with Existing Correlations

With the preceding expressions, Eq. (9) can be used in Eq. (10), yielding the results shown in Figs. 2–5 for water (μ_L = 1 cp, σ = 73.4 dyne/cm, and ρ_L = 1 g/cm^3), and diesel oil, DF-2 (μ_L = 2.61 cp, σ = 27.0 dyne/cm, and ρ_L = 0.86 g/cm^3, see Ref. 1), depicting the droplet diameter vs mass flow rate for different values of Δp_L and FN and for cone angles of 60 and 90 deg. This has been done also for ethanol (μ_L = 1.2 cp, σ = 22.75 dyne/cm, and ρ_L = 0.791 g/cm^3), kerosene (μ_L = 1.6 cp, σ = 26.0 dyne/cm, and ρ_L = 0.80 g/cm^3), and heavy fuel oil at 366 K (μ_L = 37.0 cp, σ = 21.0 dyne/cm, and ρ_L = 0.96 g/cm^3), but the results are not included here for the sake of conciseness.

Figures 2–5 also include a few experimental points for water and diesel oil, DF-2 (Ref. 1). Notice, however, that, because of the lack of precision in the copying procedure, the numerical values used here might not be the actual ones utilized by those authors. Nevertheless, they show a reasonable agreement with the present model.

It is observed that experimental results for water agree well with the developed theory for a cone angle of 90 deg. For the 60-deg cone angle, the theory agrees for higher pressures. A possible explanation for this behavior is the increased fluid velocity for higher pressures and higher cone angles. Lower

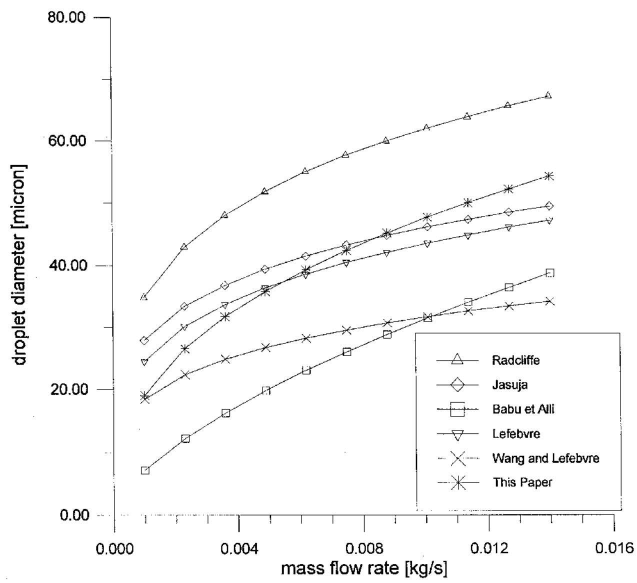

Fig. 11 Spray mean diameter vs liquid mass flow rate, for existing correlations (diesel; u = 60 deg; Dp_L = 2.07 MPa).

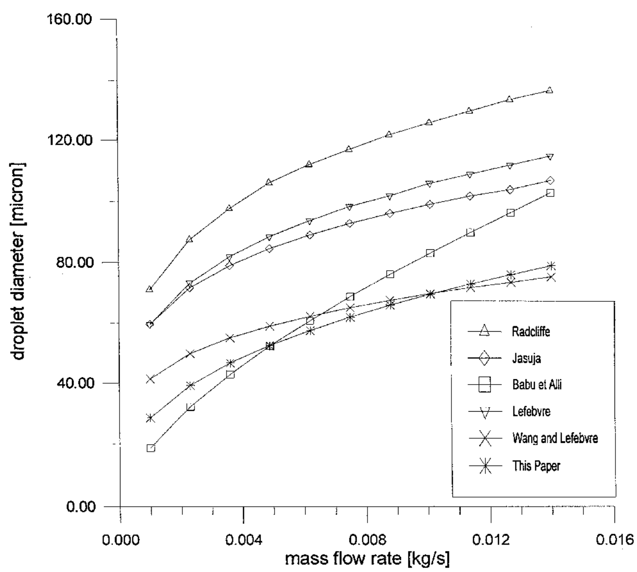

Fig. 12 Spray mean diameter vs liquid mass flow rate, for existing correlations (diesel; u = 90 deg; Dp_L = 0.35 MPa).

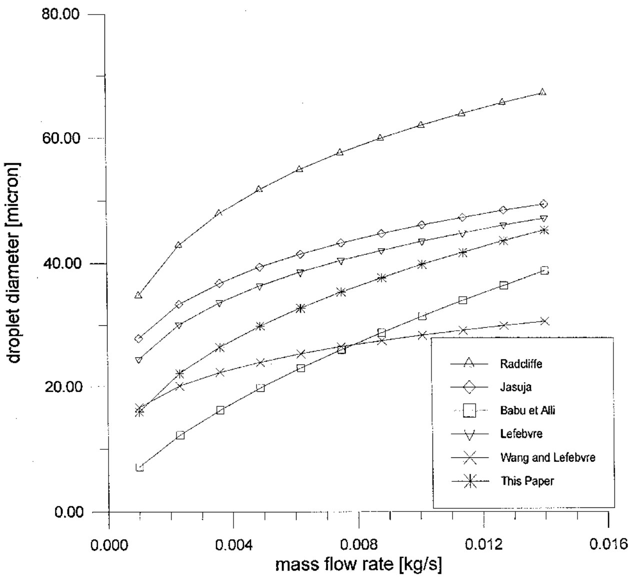

Fig. 13 Spray mean diameter vs liquid mass flow rate, for existing correlations (diesel; u = 90 deg; Dp_L = 2.07 MPa).

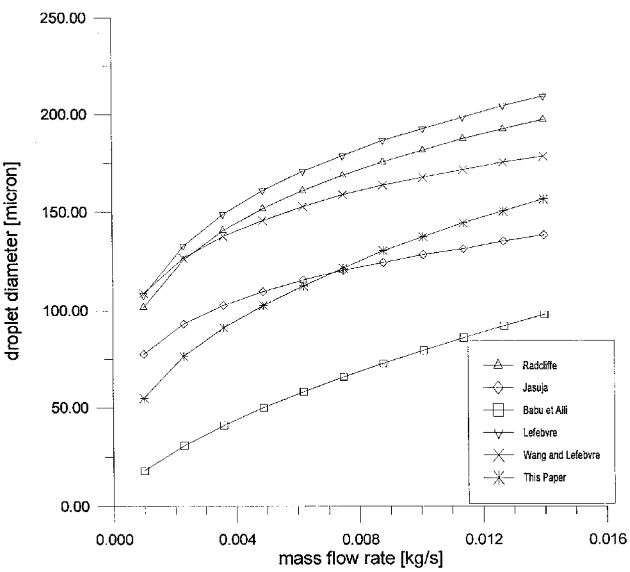

Fig. 14 Spray mean diameter vs liquid mass flow rate, for existing correlations (heavy fuel oil; u = 60 deg; Dp_L = 0.35 MPa).

pressures and angles induce coalescence, which increases the droplet size and the simplifying assumptions may no longer hold. As viscosity increases (diesel oil, Figs. 4 and 5), the problem is also noticed for lower pressures and cone angles. However, practical pressure-swirl nozzles usually operate at pressures above 7 atm (0.7 MPa) and cone angles above 60 deg, and in these ranges the developed theory is expected to work well.

The preceding results are next compared with the works of various authors,[9,15] all of them expressing surface–volume mean diameters. Figures 6–17 show this comparison for Δp_L = 0.35 and 2.07 MPa, for cone angles of 90 and 60 deg, respectively, for water, diesel oil DF-2, and heavy fuel oil at 366 K.

It is observed in Figs. 6, 10, and 14 that for a low viscosity and high surface tension liquid (water), with a cone angle of 60 deg and for a low injection pressure (0.35 MPa), the developed formulation agrees well with the semiempirical formulation of Wang and Lefebvre[1]; whereas for heavy fuel, for the same cone angle and injection pressure, agreement occurs with the Jasuja's formulation.[15] Agreement for diesel oil is only partial, with Wang and Lefebvre[1] for low flow rates, and with Jasuja[15] for high flow rates.

At higher pressures (2.07 MPa) and a cone angle of 60 deg, the present theory yields results for water between those of Wang and Lefebvre[1] and Lefebvre,[5] while for diesel oil, the results agree with those of Wang and Lefebvre[1] only for low flow rates, and with those of Jasuja[15] for higher flow rates. Thus, for 60-deg cone angles, the general behavior is independent of injection pressure, and agreement with Wang and Lefebvre[1] and Lefebvre's[5] semiempirical formulations is observed for low viscosity liquids. Agreement shifts toward Jasuja's formulation[15] as the liquid viscosity increases. In other words, this theory agrees with those semiempirical formulations that were developed for a specific liquid under given conditions, when applied to that liquid and those conditions, as expected. Jasuja,[15] for example, worked with heavy fuel oil, and his formula was applied here for water, for comparison only.

The same tendency is observed when Figs. 8, 12, and 16 and Figs. 9, 13, and 17 are analyzed as separate groups. In Figs. 8, 12, and 16, for a cone angle of 90 deg and an injection

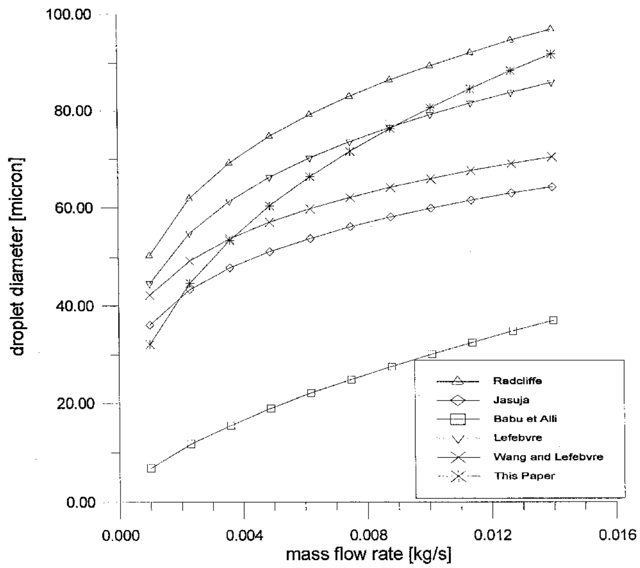

Fig. 15 Spray mean diameter vs liquid mass flow rate, for existing correlations (heavy fuel oil; u = 60 deg; Dp_L = 2.07 MPa).

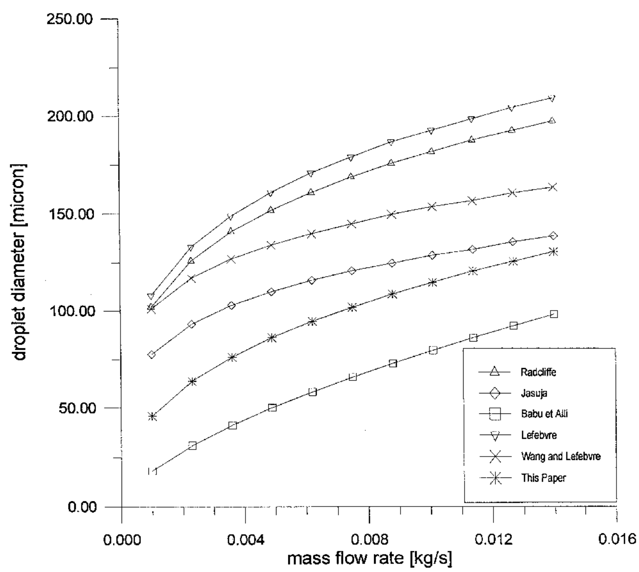

Fig. 16 Spray mean diameter vs liquid mass flow rate, for existing correlations (heavy fuel oil; u = 90 deg; Dp_L = 0.35 MPa).

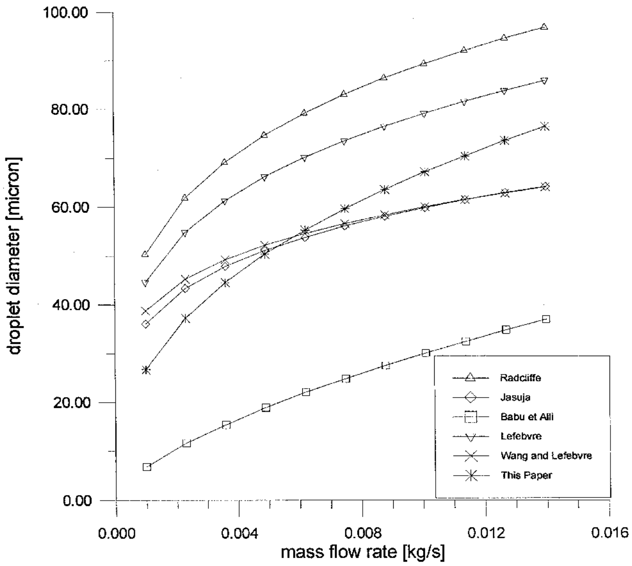

Fig. 17 Spray mean diameter vs liquid mass flow rate, for existing correlations (heavy fuel oil; θ = 90 deg; Δp_L = 2.07 MPa).

pressure of 0.35 MPa, results from the developed formulation agree well, for water and diesel oil, with results obtained using the Wang and Lefebvre's[1] formulation; whereas for heavy fuel oil, the theoretical results again tend to those of Jasuja.[15] In Figs. 9, 13, and 17, for a cone angle of 90 deg and an injection pressure of 2.07 MPa, it is observed that the theoretical results fall between the results obtained using the formulations for water,[1,9] tending to Jasuja's results[15] for heavy fuel oil, showing again the strong influence of viscosity in the expected droplet size.

Analysis of Figs. 6, 10, and 14 and Figs. 8, 12, and 16, as separate groups, shows that, for the same injection pressure, the cone angle exerts a strong influence on the droplet size. As the cone angle increases, droplet size decreases as coalescence is prevented. The same conclusion is reached analyzing Figs. 7, 11, and 15 and Figs. 9, 13, and 17.

Comparison of results of Figs. 6, 10, and 14 with those of Figs. 7, 11, and 15 shows that the droplet size decreases with an increase of the injection pressure, as expected. The same tendency is found comparing the results of Figs. 8, 12, and 16 with those of Figs. 9, 13, and 17.

The SMD equation can also be written in the form SMD = $SMD_1 + SMD_2$, as suggested by Wang and Lefebvre[1] for pressure–swirl nozzles. Because this is also the format of equations derived by Dombrowski and Johns[6] for fan sprays, by Couto and Bastos-Netto[12] and Couto et al.[16] for impinging jets, and Couto and Bastos-Netto[17] for splash plate jets, apparently this is the general format of equations for atomizers that possess liquid films.

Conclusions

A theoretical formula for estimating the droplets mean diameter, based on an hypothesis regarding the thickness of a plane disintegrating liquid sheet, was obtained for a pressure-swirl atomizer. A conical surface was considered as the disintegrating sheet.

The derived theoretical equation for the conical spray droplets mean diameter includes atomizer geometrical parameters as well as liquid fuel and atomizing gas flow parameters, and it compares well with available results from several authors.

Acknowledgments

This work was supported by the Foundation of Support for Research of the State of São Paulo (FAPESP), Brazil, under Grant 95/0455-8. The authors gratefully acknowledge Leon Sinay for his important suggestions, and Wilson F. Muniz for his assistance in the preparation of the graphs.

References

[1]Wang, X. F., and Lefebvre, A. H., "Mean Drop Sizes from Pressure-Swirl Nozzles," *Journal of Propulsion and Power*, Vol. 3, No. 1, 1987, pp. 11–18.

[2]Suyari, M., and Lefebvre, A. H., "Film Thickness Measurements in a Simplex Swirl Atomizer," *Journal of Propulsion and Power*, Vol. 2, No. 6, 1986, pp. 528–533.

[3]Rizk, N. K., and Lefebvre, A. H., "Spray Characteristics of Spill-Return Atomizers," *Journal of Propulsion and Power*, Vol. 1, No. 3, 1985, pp. 200–204.

[4]Ortman, J., and Lefebvre, A. H., "Fuel Distributions from Pressure-Swirl Atomizers," *Journal of Propulsion and Power*, Vol. 1, No. 1, 1985, pp. 11–15.

[5]Lefebvre, A. H., "The Prediction of Sauter Mean Diameter for Simplex Pressure-Swirl Atomizers, *Atomization Spray Technology*, Vol. 3, No. 1, 1987, pp. 37–51.

[6]Dombrowski, N., and Johns, W. R., "The Aerodynamic Instability and Disintegration of Viscous Liquid Sheets," *Chemical Engineering Science*, Vol. 18, No. 2, 1963, pp. 203–214.

[7]Fraser, R. P., Eisenklam, P., Dombrowski, N., and Hasson, D., *AIChE Journal*, Vol. 8, No. 5, 1962, pp. 672–680.

[8]Taylor, G. I., "Formation of Thin Flat Sheets of Water," *Proceedings of the Royal Society of London*, Vol. 259A, No. 1195, 1960, pp. 1–17.

[9]Lefebvre, A. H., *Atomization and Sprays*, Hemisphere, New York, 1989.

[10]Eisenklam, P., "Recent Research and Development Work on Liquid Atomization in Europe and the USA," 5th Conf. on Liquid Atomization, Tokyo, 1976.

[11]York, J. L., Stubbs, H. F., and Tek, M. R., "The Mechanism of Disintegration of Liquid Sheets," *Transactions of the American Society of Mechanical Engineers*, Vol. 75, 1953, pp. 1279–1286.

[12]Couto, H. S., and Bastos-Netto, D., "Modeling Droplet Size Distribution from Impinging Jets," *Journal of Propulsion and Power*, Vol. 7, No. 4, 1991, pp. 654–656.

[13]Rizk, N. K., and Lefebvre, A. H., "Internal Flow Characteristics of Simplex Swirl Atomizers, *Journal of Propulsion and Power*, Vol. 1, No. 3, 1985, pp. 193–199.

[14]Simmons, H. C., and Harding, C. F., "Some Effects of Using Water as a Test Fluid in Fuel Nozzle Spray Analysis," American Society of Mechanical Engineers, Paper 80-GT-90, March 1980.

[15]Jasuja, A. K., Atomization of Crude and Residual Fuel Oils, *Journal of Engineering for Power*, Vol. 101, No. 2, 1979, pp. 250–258.

[16]Couto, H. S., Bastos-Netto, D., and Migueis, C. E., "Construction of the Droplet Size Distribution Function in the Spray Formed by Impinging Jets," *Journal of Propulsion and Power*, Vol. 8, No. 3, 1992, pp. 725–728.

[17]Couto, H. S., and Bastos-Netto, D., "Theory and Design Technique a Splash Plate Atomizer," *Proceedings of the IV Brazilian Thermal Sciences Meeting*, Brazilian Society of Mechanical Sciences, Rio de Janeiro, 1992, pp. 57, 58.